Semiconjugacy to a map of a constant slope

Michał Misiurewicz

Indiana University - Purdue University Indianapolis
Department of Mathematical Sciences
Indianapolis, IN, USA
mmisiure@math.iupui.edu
http://www.math.iupui.edu/∼mmisiure

In the book [1] there is a proof that every continuous piecewise monotone interval map with positive entropy is semiconjugate to a map with constant slope. This proof is different from the classical proof of Milnor and Thurston. It turns out that a small modification of this proof works for piecewise continuous piecewise monotone interval maps. As a corollary we get a version of this theorem for (piecewise) continuous piecewise monotone graph maps.

The main idea of the proof is to approximate the map with the Markov ones. For Markov maps the semiconjugacy can be constructed easily using symbolic dynamics. Then we want to get our semiconjugacy by passing to the limit. However, it can happen that consecutive better and better approximations produce semiconjugacies that blow up smaller and smaller intervals, collapsing to a finite set larger and larger portions of the interval. To avoid this situation, we construct the approximations with the same laps of high iterates as for the original map.

To pass from piecewise continuous interval maps to graph maps, we cut the graph into pieces at the vertices and first preimages of the vertices, and then put those pieces together to get an interval. This process introduces new discontinuities, but if the graph map was piecewise monotone, there will be only finite number of them. Therefore we can apply the theorems on piecewise continuous piecewise monotone interval maps to (piecewise) continuous piecewise monotone graph maps.

The semiconjugacies that we consider are nondecreasing surjections. They may collapse some intervals to points. For graphs, this means that some nontrivial portions of the graph can be collapsed to the points, so the image of the graph can be a different graph.

This problem does not exist if the map is transitive. Then the semiconjugacy is a conjugacy. In particular, the graph we get after the semiconjugacy is homeomorphic to the original one.

References and Literature for Further Reading

[1] Ll. Alsedà, J. Llibre and M. Misiurewicz, "Combinatorial Dynamics and Entropy in Dimension One" (Second Edition), Advanced Series in Nonlinear Dynamics vol. **5**, World Scientific, Singapore, 2000.